Neutrino in magnetic fields: from the first studies to the new effects in neutrino oscillations

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Abstract

In this paper I should like to present **the four new effects** in neutrino oscillations that have been recently investigated in my research group at the Department of Theoretical Physics of the Moscow State University. Due to the fact that these studies were stimulated by our previous research of neutrino interactions in the presence of magnetic fields, and also because the year 2004 commemorates the 40th years jubilee since the the first paper on the neutrino interaction in a magnetic field was published, a short review on the first papers dedicated to the problem of neutrinos in magnetic fields, and also on the recent results in this field, prefaces (Section 1) the discussion on **the new effects** in neutrino oscillations. Section 2 is devoted to our recent studies of the electromagnetic properties of a *massive* neutrino, including the neutrino magnetic moment for different values of neutrino mass. In Section 3 we discuss **the four new effects** in neutrino spin and flavour oscillations in different background environments.

1   Beta-decay of neutron in magnetic field

About forty years ago the first studies of the neutrino interaction in the presence of a magnetic field were performed in the two papers [1] dedicated to the polarized neutron beta-decay $n \rightarrow p + e + \tilde{\nu}_e$ in a magnetic field. In these papers the probability of the polarized neutron beta-decay in the presence of a magnetic field was derived, as well as the asymmetry in the neutrino emission was studied for the first time. It was shown that the differential rate of the process exhibits the resonance spikes which appears, for the given magnetic field strength, each time when the final electron energy is exactly equal to one of the allowed Landau energies in the magnetic field. It was also shown that the

total rate depends on the initial neutron polarization, in contrast with the field-free case when the neutron polarization dependence disappears from the rate during integration over the phase space of the process. The range of magnetic field strengths considered in these papers span up to subcritical fields $B \geq B_0 = \frac{m^2}{e} = 4.41 \times 10^{13} \ Gauss$. It worth to be noted here that these studies were performed before the discovery of pulsars [2] where such a strong magnetic fields are believed to exist.

In the two well known papers [3], published a few years later, the results of [1] for the neutron decay rate in a magnetic field were re-derived and the problem of the neutron star cooling was considered. However there were no discussion on the asymmetry in neutrino emission in the papers of ref.[3].

Very strong magnetic fields are also supposed to exist in the early Universe (for a recent review see, e.g. [4]). As it was discussed for the first time in [5], the weak reaction rates of the processes like

$$n \rightarrow p + e + \tilde{\nu}_e, \quad \nu_e + n \leftrightharpoons e + p, \quad p + \tilde{\nu}_e \leftrightharpoons n + e^+, \tag{1}$$

which determine the iter-conversion between neutrons and protons and set the n/p-ration in various environments, can be significantly modified under the influence of magnetic fields and, as a consequence, influence the primordial nucleosynthesis affecting production of ^4He .

The aforementioned studies of neutrino interactions in the presence of magnetic fields [1, 3, 5] gave the birth to **the neutrino astrophysics in magnetic fields**.

Various authors (the first papers in this field are given in ref. [6]) argued that asymmetric neutrino emission in the direct URCA processes (1) during the first seconds after the massive star collapse could provide explanations for the observed pulsar velocities. We have shown [7] that in order to get a correct prediction for the direction and value of the kick velocity of a pulsar one has to account not only for the amount of radiated in the processes (1) neutrinos but also for the fact that the average momentum of neutrinos propagating in the opposite directions are not equal one to each other. Some of the recent studies in this field can be found in [8]. A lot of other different mechanisms for the asymmetry in the neutrino emission from a magnetized pulsar has been also studied previously (see e.g. [9]). For more complete references to the performed studies on the neutrino mechanisms of the pulsar kicks see the second review paper of ref.[10].

Recently we have developed [11] the relativistic approach to the inverse β-decay of a polarized neutron, $\nu_e + n \rightarrow p + e^-$, in a magnetic field. This process can be also important for the neutrino transport inside the magnetized pulsar and contribute to the kick velocities [10]. As we have shown [11], in strong magnetic fields the cross section can be highly anisotropic in respect to the neutrino angle. In the particular case of polarized neutrons, matter becomes even transparent for neutrinos if neutrinos propagate against the direction of neutrons polarization.

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2 Electromagnetic properties of a massive neutrino [1, 2]

It is well known that neutrino with non-zero mass has non-trivial electromagnetic properties. In particular, the Dirac massive neutrino can posses non-vanishing magnetic and electric dipole moments. Noted here that the massive Majorana neutrino can't have neither magnetic no electric moments in vacuum. However, the Majorana neutrino can have flavour non-diagonal (transition) magnetic and electric moments. It is believed that non-zero neutrino magnetic moment could have an important impact on astrophysics and cosmology.

It is also well known [3] that in the minimally extended Standard Model with $SU(2)$-singlet right-handed neutrino the one-loop radiative correction generates neutrino magnetic moment which is proportional to the neutrino mass

$$\mu_\nu = \frac{3}{8\sqrt{2}\pi^2}eG_F m_\nu = 3 \times 10^{-19}\mu_0\left(\frac{m_\nu}{1\text{eV}}\right), \tag{2}$$

where $\mu_0 = e/2m$ is the Bohr magneton, m_ν and m are the neutrino and electron masses. There are also models [4] in which much large values for magnetic moments of neutrinos are predicted. So far, the most stringent laboratory constraints on the electron, muon, and tau neutrino magnetic moments come from elastic neutrino-electron scattering experiments: $\mu_{\nu_e} \leqslant 1.5 \times 10^{-10}\mu_0$, $\mu_{\nu_\mu} \leqslant 6.8 \times 10^{-10}\mu_0$, $\mu_{\nu_\tau} \leqslant 3.9 \times 10^{-10}\mu_0$, [5].

Recently we have considered [1, 2] the massive Dirac neutrino electromagnetic form factors in the context of the standard model supplied with SU(2)-singlet right-handed neutrino. Using the dimensional-regularization scheme, we have performed the most general study of the *massive* neutrino one-loop vertex function in the general R_ξ gauge exactly accounting for the masses of all particles in polarization loops. In [2] we have also accounted for neutrino mixing effects and discussed the massive neutrino anapole moment in details.

The neutrino electromagnetic vertex function $\Lambda_\mu(q)$ is shown in Fig. 1.
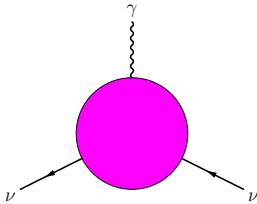
The one-loop contributions to the neutrino electromagnetic vertex $\Lambda_\mu(q)$ are given by the two types of Feynman diagrams: the proper vertices [Fig. 3(a)-3(f)] and the $\gamma-Z$ self-energy diagrams [Fig. 4(a)-4(h)].

The matrix element of the electromagnetic current between neutrino states can be presented in the form
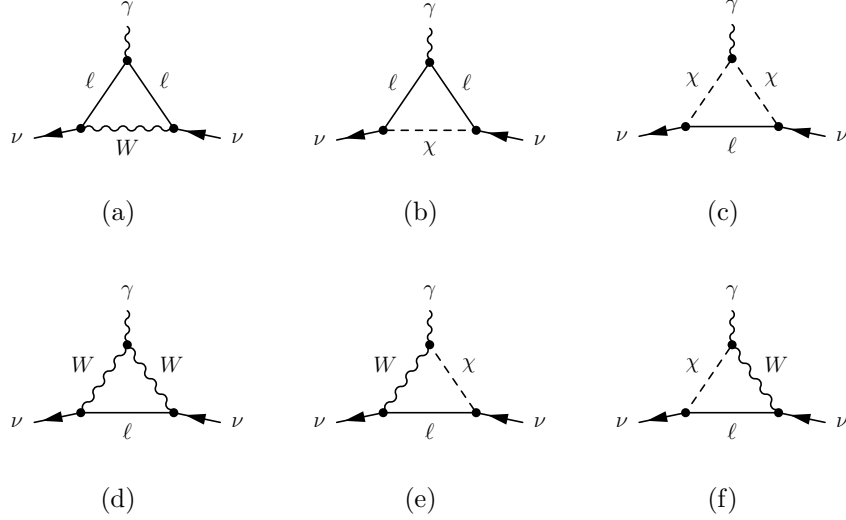

Figure 1: Neutrino electromagnetic vertex function

Figure 3: (a)-(f) proper vertices.

$$\langle \nu(p')|J_\mu^{\mathrm{EM}}|\nu(p)\rangle = \bar{u}(p')\Lambda_\mu(q)u(p),$$

where the most general expression for the electromagnetic vertex function $\Lambda_\mu(q)$ reads

$$\Lambda_\mu(q) = f_Q(q^2)\gamma_\mu + f_M(q^2)i\sigma_{\mu\nu}q^\nu - f_E(q^2)\sigma_{\mu\nu}q^\nu\gamma_5 + f_A(q^2)(q^2\gamma_\mu - q_\mu \slashed{q})\gamma_5.$$

Here $f_Q(q^2)$, $f_M(q^2)$, $f_E(q^2)$ and $f_A(q^2)$ are respectively the electric, dipole electric, dipole magnetic, and anapole neutrino form factors, $q_\mu = p'_\mu - p_\mu$, $\sigma_{\mu\nu} = (i/2)[\gamma_\mu, \gamma_\nu]$, $\gamma_5 = -i\gamma^0\gamma^1\gamma^2\gamma^3$.

The contributions of the $\gamma - Z$ diagrams are schematically presented in Fig. 2. Performing the direct calculations of the loop diagrams we established that:

1) neutrino electromagnetic function consist of only three electromagnetic form factors (in the case of a model with CP conservation), 2) there is such a gauge in which all electromagnetic form factors are finite, i.e. they do not contain ultraviolet divergences. The values of the gauge fixing parameters are

$$\alpha_W = \frac{1}{9}(138 + 151\tan^2\theta_W), \quad \alpha_Z = +\infty \qquad (3)$$
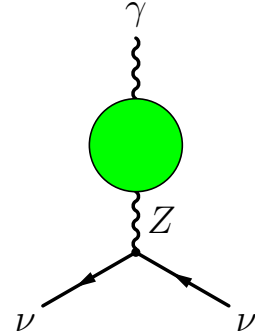
We also found [1, 2] the closed integral expressions for electric, magnetic, and anapole form factors of a *massive* neutrino. On this basis we have derived the electric charge

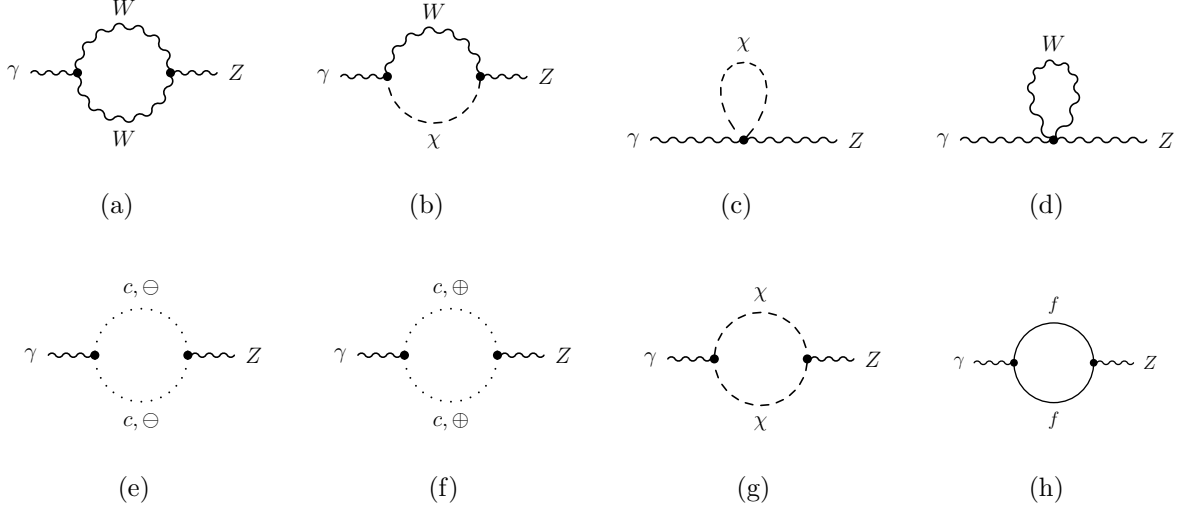Figure 2: The contributions of $\gamma - Z$ diagrams

Figure 4: (a)-(h) The $\gamma - Z$ self-energy diagram. f denotes the electron, muon, and τ-lepton as well as u, c, t, d, s, and b quarks.

(the value of the electric form factor at zero momentum transfer), magnetic moment, and anapole moment of a *massive* neutrino. By means of the direct calculations for the case of a *massive* neutrino we have shown that: 1) electric charge is independent of the gauge parameters and is equal to zero, 2) magnetic moment does not depend on the choice of the gauge.

The recent LEP data require that the number of light neutrinos coupling to Z boson is exactly three, whereas any additional neutrino, if this particle exist, must be heavy. That is the reason to consider the neutrino magnetic moment for various ranges of particles masses. We have obtained [1] the values of the neutrino magnetic moment for light (for this particular case see also [3, 6]), intermediate and heavy massive neutrino:

$$1)\ \ m_\nu \ll m_\ell \ll M_W,$$

$$\mu_\nu = \frac{eG_F}{4\pi^2\sqrt{2}}m_\nu\frac{3}{4(1-a)^3}(2 - 7a + 6a^2 - 2a^2\ln a - a^3),\quad a = \Big(\frac{m_l}{M_W}\Big)^2,$$

$$2)\ \ m_\ell \ll m_\nu \ll M_W,$$

$$\mu_\nu = \frac{3eG_F}{8\pi^2\sqrt{2}}m_\nu\left\{1 + \frac{5}{18}b\right\},\quad b = \Big(\frac{m_\nu}{M_W}\Big)^2,$$

$$3)\ \ m_\ell \ll M_W \ll m_\nu,$$

$$\mu = \frac{eG_F}{8\pi^2\sqrt{2}}m_\nu.$$

In the conclusion of this section, we should like to note that the neutrino electromagnetic properties are affected by the external environment. In particular, a neutrino can acquire

an electric charge in a magnetized matter [7], also the value of the neutrino magnetic moment can be significantly shifted by the presence of strong external magnetic fields [8, 9]. The recent study of the neutrino electromagnetic vertex in a magnetized matter can be found in [10].

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3 Four new effects in neutrino oscillations in background environments [8, 9, 11, 12, 13, 15, 16, 17, 18]

In this section we present **the four new effects** in neutrino oscillations that have been recently studied in our papers [8, 9], [11]- [13], and [15]- [18].

The thorough research on neutrino oscillations were launched by the pioneering paper of Bruno Pontecorvo [1]. Indeed, it is surprising that after more than 45 years of activity in this field (for some of the most important papers see refs.[2, 3, 4, 5, 6]) it is still possible to discover new effects.

The whole story began several years ago when we studied neutrino spin oscillations in strong magnetic fields [7] and to our much surprise realized that at that time in literature there were no even attempts to consider neutrino spin oscillations in any electromagnetic field configuration rather than constant in time and transversal in respect to neutrino motion magnetic fields (see, for example, [6]). Further more, in all of the studies of neutrino spin and also flavour oscillations in matter, performed before 1995, the matter effect was treated only in the non-relativistic limit (matter was always supposed to be slowly moving or to be at rest, see for example [4]).

The first our attempt [8] to consider neutrino flavour oscillations in matter in the case when matter is moving with relativistic speed was made in 1995. In that our study we have tried to apply the Lorentz invariant formalism to describe neutrino flavour oscillations and realized that the value of the matter term in the neutrino effective potential can be significantly changed if matter is moving with relativistic speed. In particular, in [8] we have shown that the difference of neutrino effective potentials, V_{eff}, in non-polarized relativistically moving matter composed of electrons is proportional to $V_{eff} \sim (1 - \vec{\beta}\vec{v}_e)$, where $\vec{\beta}$ and $\vec{v}_e$ are the speeds of neutrino and electrons, correspondingly. Thus, we have observed in 1995 that, for the case of matter moving with relativistic speed along the direction of the ultra relativistic neutrino propagation, the effect of matter in oscillations is washed out.

We have continued [9] our studies on evaluation of the Lorentz invariant formalism in neutrino oscillations in 1999 when have developed an approach that enables us to consider neutrino spin oscillations in arbitrary electromagnetic field configurations. We have shown [9] that the Bargmann-Michel-Telegdi equation [10], describing a neutral particle spin evolution under the influence of an electromagnetic field, can be generalized for the case of a neutrino moving in electromagnetic fields and matter by implementing the substitution of the external electromagnetic field tensor, $F_{\mu\nu} = (\vec{E}, \vec{B})$, according to the prescription $F_{\mu\nu} \rightarrow F_{\mu\nu} + G_{\mu\nu}$. The anti-symmetric tensor $G_{\mu\nu} = (-\vec{P}, \vec{M})$ can be constructed with use of the neutrino speed, matter speed, and matter polarization four-vectors under the natural assumptions that the neutrino spin evolution equation have to be linear over $F_{\mu\nu}$ and over the other mentioned above vectors. From this new generalized BMT equation for the neutrino spin evolution in an electromagnetic field and matter we have finally arrive to

the following equation for the evolution of the three-dimensional neutrino spin vector $\vec{S}$:

$$\frac{d\vec{S}}{dt} = \frac{2\mu}{\gamma}\Big[\vec{S} \times (\vec{B}_0 + \vec{M}_0)\Big], \tag{4}$$

$$\vec{B}_0 = \gamma\Big(\vec{B}_\perp + \frac{1}{\gamma}\vec{B}_\parallel + \sqrt{1-\gamma^{-2}}\Big[\vec{E}_\perp \times \vec{n}\Big]\Big), \ \gamma = (1-\beta^2)^{-\frac{1}{2}}, \tag{5}$$

$$\vec{M}_0 = \vec{M}_{0_\parallel} + \vec{M}_{0_\perp}, \tag{6}$$

$$
\begin{aligned}
\vec{M}_{0_\parallel} = \gamma\vec{\beta}\frac{n_0}{\sqrt{1-v_e^2}}&\left\{\rho_e^{(1)}\left(1 - \frac{\vec{v}_e\vec{\beta}}{1-\gamma^{-2}}\right)\right.\\
&\left.-\rho_e^{(2)}\left(\vec{\zeta}_e\vec{\beta}\sqrt{1-v_e^2} + \frac{(\vec{\zeta}_e\vec{v}_e)(\vec{\beta}\vec{v}_e)}{1+\sqrt{1-v_e^2}}\right)\frac{1}{1-\gamma^{-2}}\right\},
\end{aligned}
\tag{7}
$$

$$\vec{M}_{0_\perp} = -\frac{n_0}{\sqrt{1-v_e^2}}\left\{\vec{v}_{e\perp}\left(\rho_e^{(1)} + \rho_e^{(2)}\frac{(\vec{\zeta}_e\vec{v}_e)}{1+\sqrt{1-v_e^2}}\right) + \vec{\zeta}_{e\perp}\rho_e^{(2)}\sqrt{1-v_e^2}\right\}, \tag{8}$$

where t is time in the laboratory frame, $\vec{F}_\perp$ and $\vec{F}_\parallel$ ($\vec{F} = \vec{B}, \vec{E}$) are transversal and longitudinal (with respect to the direction $\vec{n}$ of neutrino motion) electromagnetic field components in the laboratory frame. For simplify we neglect here the neutrino electric dipole moment, $\epsilon = 0$, and also consider the case when matter is composed of only one type of fermions (electrons). The general case of $\epsilon \neq 0$ and matter composed of different types of leptons is discussed in our papers [15, 16]. Here $n_0 = n_e\sqrt{1-v_e^2}$ is the invariant number density of matter given in the reference frame for which the total speed of matter is zero. The vectors $\vec{v}_e$, and $\vec{\zeta}_e$ ($0 \leqslant |\vec{\zeta}_e|^2 \leqslant 1$) denote, respectively, the speed of the reference frame in which the mean momentum of matter (electrons) is zero, and the mean value of the polarization vector of the background electrons in the above mentioned reference frame. The coefficients $\rho_e^{(1,2)}$ are calculated if the neutrino Lagrangian is given, and within the extended standard model supplied with $SU(2)$-singlet right-handed neutrino ν_R,

$$\rho_e^{(1)} = \frac{\tilde{G}_F}{2\sqrt{2}\mu}, \qquad \rho_e^{(2)} = -\frac{G_F}{2\sqrt{2}\mu}, \tag{9}$$

where $\tilde{G}_F = G_F(1 + 4\sin^2\theta_W)$.

3.1 Neutrino spin oscillations in electromagnetic fields

From the new generalized BMT equation [9] for the neutrino spin evolution in an electromagnetic field and matter (and also from the simplified version of the equation given by (4)) the corresponding Hamiltonians describing neutrino spin oscillations are just straightforward. Thus, **the first new effect** is the prediction of neutrino spin oscillations in

various electromagnetic field configurations. We have derived [9, 11, 17] the new resonances in neutrino oscillations for several electromagnetic fields such as the field of circular and linearly polarized electromagnetic waves and superposition of an electromagnetic wave and constant magnetic field. We have also studied (see the second paper of ref. [11]) the possibility for parametric resonances in neutrino oscillations in periodically varying electromagnetic fields (an electromagnetic wave with varying amplitude and a "castle wall" magnetic field of an undulator).

3.2 Neutrino spin oscillations in matter and different external fields

We have predicted [9, 12], using the generalized BMT equation, **the second new effect**: neutrino spin procession can be stimulated not only by presence of electromagnetic fields (i.e., by electromagnetic interactions of neutrino) but also by weak interactions of neutrino with background matter. Thus, neutrino spin procession could appear without any electromagnetic field in the presence of matter through which neutrino propagates. This conclusion is just straightforward also from the simplified version of the spin evolution equation (4).

Moreover, we have shown [16] that the neutrino spin precession *always occurs* in presence of matter (even in the case of non-moving and unpolarized matter) if the initial neutrino state is not longitudinally polarized. Indeed, as it follows from (4), in the laboratory reference frame the corresponding equation for the three-dimensional neutrino spin is

$$\frac{d\vec{S}}{dt} = 2\mu\rho_e^{(1)}n_e[\vec{S} \times \vec{\beta}]. \tag{10}$$

If neutrino is propagating along the OZ axis, $\vec{\beta} = (0, 0, \beta)$, then solutions of these equations for the neutrino spin components are given by

$$S^1 = S_0^\perp \cos\omega t, \ \ S^2 = S_0^\perp \sin\omega t, \ \ S^3 = S_0^3, S^0 = S_0^0, \tag{11}$$

where

$$\omega = 2\mu\rho_e^{(1)}n_e\beta, \tag{12}$$

$S_0^\perp$ and $S_0^{3,0}$ are constants determined by the initial conditions. Obviously, if $S_0^\perp \neq 0$ then the solution of eq.(10) is not trivial (non-zero) and there is a procession of the neutrino spin vector $\vec{S}$ in the background matter.

Within the developed Lorentz invariant approach it is also possible [13] to find the solution for the neutrino spin evolution problem in a more general case when the neutrino is subjected to general types of non-derivative interactions with external fields that are given by the Lagrangian

$$-\mathcal{L} = g_s s(x)\bar{\nu}\nu + g_p \pi(x)\bar{\nu}\gamma^5\nu + g_v V^\mu(x)\bar{\nu}\gamma_\mu\nu + g_a A^\mu(x)\bar{\nu}\gamma_\mu\gamma^5\nu + \frac{g_t}{2}T^{\mu\nu}\bar{\nu}\sigma_{\mu\nu}\nu + \frac{g_t'}{2}\Pi^{\mu\nu}\bar{\nu}\sigma_{\mu\nu}\gamma_5\nu, \tag{13}$$

where $s, \pi, V^\mu = (V^0, \vec{V}), A^\mu = (A^0, \vec{A}), T_{\mu\nu} = (\vec{a}, \vec{b}), \Pi_{\mu\nu} = (\vec{c}, \vec{d})$ are the scalar, pseudoscalar, vector, axial-vector, tensor, pseudotensor fields, respectively. For the neutrino spin evolution equation in this case we have found

$$
\begin{aligned}
\frac{d\vec{S}}{dt} = {} & 2g_a \left\{ A^0[\vec{S} \times \vec{\beta}] - \frac{1}{1+\gamma^{-1}}(\vec{A}\vec{\beta})[\vec{S} \times \vec{\beta}] - \frac{1}{\gamma}[\vec{S} \times \vec{A}] \right\} \\
& + 2g_t \left\{ [\vec{S} \times \vec{b}] - \frac{1}{1+\gamma^{-1}}(\vec{\beta}\vec{b})[\vec{S} \times \vec{\beta}] + [\vec{S} \times [\vec{a} \times \vec{\beta}]] \right\} \\
& + 2ig_t' \left\{ [\vec{S} \times \vec{c}] - \frac{1}{1+\gamma^{-1}}(\vec{\beta}\vec{c})[\vec{S} \times \vec{\beta}] - [\vec{S} \times [\vec{d} \times \vec{\beta}]] \right\}.
\end{aligned}
\tag{14}
$$

It is worth to be noted that (see also [14]) neither scalar nor pseudoscalar nor vector interaction contributes to the neutrino spin evolution.

The neutrino spin evolution equation (14) can be used for any theoretical model in which neutrino has mentioned above general interactions. For instance, within the standard model the weak interaction of a neutrino with the background matter has the axial vector term, whereas the electromagnetic interaction of a neutrino is described by the tensor field $T_{\mu\nu} = F_{\mu\nu} = (\vec{E}, \vec{B})$. As it has been also recently shown [18], the considered general equation (14) can be used for description of the neutrino spin oscillations in a gravitational field of a rotating object. In this case, the gravitational field in the weak-field limit can be treated as an external axial vector field.

3.3 Relativistic matter motion effects in neutrino oscillations

The derived new equation for the neutrino spin evolution enables us to study spin oscillations in the case of moving with arbitrary (also relativistic) speed and polarized matter. We have predicted [12] **the third effect**: the matter motion can drastically change the neutrino oscillation pattern and, in particular, can significantly shift the neutrino spin oscillation resonance condition (see the third and fourth papers of ref. [6]), if compared with the case of non-moving matter.

We use again the simplified neutrino spin evolution equation (4). In the case of slowly moving matter, $v_e \ll 1$, from eqs. (6), (7), and (8) we get

$$
\vec{M}_0 = n_e \gamma \vec{\beta} \left(\rho_e^{(1)} - \rho_e^{(2)} \vec{\zeta}_e \vec{\beta} \right),
\tag{15}
$$

in agreement with results of [4, 20]. In the opposite case of relativistic flux, $v_e \sim 1$, we find,

$$
\vec{M}_0 = \frac{n_0}{\sqrt{1-v_e^2}} \left(\rho_e^{(1)} + \rho_e^{(2)} \vec{\zeta}_e \vec{v}_e \right) \left(1 - \vec{\beta}\vec{v}_e \right).
\tag{16}
$$

One can easily see that the matter effect can be annihilated owing to the relativistic motion of matter along the direction of neutrino propagation, provided that $1 - \vec{\beta}\vec{v}_e \approx 0$. We also predict significant increase of matter effect in neutrino spin oscillations for neutrino propagating against the relativistic flux of matter.

To illustrate this phenomenon let us consider the case of neutrino spin oscillations in the flux of electrons, that could move with arbitrary (also relativistic) speed, under the influence of an arbitrary constant magnetic field, $\vec{B} = \vec{B}_{\parallel} + \vec{B}_{\perp}$. In the adiabatic approximation for the particular case of electron neutrinos ν_e propagating in matter composed of electrons, the probability of conversion $\nu_L \to \nu_R$ can be written in the form [12],

$$P_{\nu_L \to \nu_R}(x) = \sin^2 2\theta_{eff} \sin^2 \frac{\pi x}{L_{eff}}, \tag{17}$$

$$sin^2 2\theta_{eff} = \frac{E_{eff}^2}{E_{eff}^2 + \Delta_{eff}^2}, L_{eff} = \frac{2\pi}{\sqrt{E_{eff}^2 + \Delta_{eff}^2}}, \tag{18}$$

where $E_{eff} = 2\mu B_{\perp}$ (terms $\sim O(\gamma^{-1})$ are omitted here), and

$$\Delta_{eff} = V(1 - \vec{\beta}\vec{v}_e) + \frac{2\mu B_{\parallel}}{\gamma}, \quad V = \frac{G_F}{\sqrt{2}} \frac{n_0}{\sqrt{1 - v_e^2}}(1 + 4\sin^2 \theta_W). \tag{19}$$

As it is mentioned above, the matter effect in Δ_{eff} can be "eaten" by the relativistic motion of matter if $(1 - \vec{\beta}\vec{v}_e) \approx 0$. In the case of the neutrino and matter relativistic motion (β and $v_e \sim 1$) in opposite directions $(1 - \vec{\beta}\vec{v}_e \approx 2)$, the matter term contribution V can be reasonably increased due to the presence of a small term $\sqrt{1 - v_e^2}$ in the dominator.

The analogous **effect** also exist [8, 15] in neutrino flavour oscillations: the neutrino resonance condition [4, 5] can be significantly modified if matter is moving with relativistic speed. The probability of neutrino conversion $\nu_e \to \nu_\mu$ in arbitrary moving and polarized matter (composed, for instance, only of electrons) can be written in the form

$$P_{\nu_e \to \nu_\mu}(x) = \sin^2 2\theta_{eff} \sin^2 \frac{\pi x}{L_{eff}}, \tag{20}$$

where the effective mixing angle, θ_{eff}, and the effective oscillation length, L_{eff}, are given by [15]

$$\sin^2 2\theta_{eff} = \frac{\Delta^2 \sin^2 2\theta}{\left(\Delta \cos 2\theta - A\right)^2 + \Delta^2 \sin^2 2\theta}, L_{eff} = \frac{2\pi}{\sqrt{\left(\Delta \cos 2\theta - A\right)^2 + \Delta^2 \sin^2 2\theta}}. \tag{21}$$

Here $\Delta = \delta m_\nu^2 / 2|\vec{p}|$ and $\vec{p}$ is the neutrino momentum, θ is the vacuum mixing angle and

$$A = \sqrt{2} G_F \frac{n_0}{\sqrt{1 - v_e^2}} \left\{ (1 - \vec{\beta}\vec{v}_e)(1 - \vec{\zeta}_e \vec{v}_e) + \sqrt{1 - v_e^2} \left[\vec{\zeta}_e \vec{\beta} - \frac{(\vec{\beta}\vec{v}_e)(\vec{\zeta}_e \vec{v}_e)}{1 + \sqrt{1 - v_e^2}} \right] \right\}. \tag{22}$$

One can see that the neutrino oscillation probability, $P_{\nu_e \to \nu_\mu}(x)$, the mixing angle, θ_{eff}, and the oscillation length, L_{eff}, exhibit dependence on the total speed of electrons $\vec{v}_e$, correlation between $\vec{\beta}$, $\vec{v}_e$ and polarization of matter $\vec{\zeta}_e$. The resonance condition

$$\frac{\delta m_\nu^2}{2|\vec{p}|} \cos 2\theta = A, \tag{23}$$

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at which the probability has unit amplitude, also depends on the motion and polarization of matter and neutrino speed. It follows that the relativistic motion of matter could provide appearance of (destroy) the resonance in the neutrino oscillations in certain cases when for the given neutrino characteristics, δm_ν^2, $|\vec{p}|$ and θ, and the invariant matter density at rest, n_0, the resonance is impossible (exists). A detailed analysis of the neutrino effective potential in moving and polarized matter for different particular cases (for different speeds and polarizations of matter) can be found in [15].

3.4 Spin light of neutrino in background environments

The **fourth new effect** in neutrino oscillations is the prediction [16, 18] for the new mechanisms of electromagnetic radiation by neutrino moving in background matter and/or electromagnetic and gravitational fields. The new mechanism of electromagnetic radiation (we have named [16] this radiation *"spin light of neutrino"* $(SL\nu)$) originates from the neutrino spin precession that can be produced whether by weak interactions with matter or by interactions with external electromagnetic fields [16], or by interactions with gravitational fields [18]. This radiation in the case when the neutrino spin precession is induced by a constant magnetic field was also considered before in [19].

The total power of the $SL\nu$ does not washed out even when the emitted photon refractive index in the background matter is equal to unit. That is why the $SL\nu$ can not be considered as the neutrino Cerenkov radiation (see, for example, [21] and references therein).

If we assume, for definiteness, that the spin light radiation is produced by the electron neutrino ν_e moving in unpolarized ($\zeta_e = 0$) matter composed of only electrons (the more general cases are also considered in [16]) and constant magnetic field $\vec{B} = \vec{B}_\perp + \vec{B}_\parallel$ then for the total spin light radiation power we get

$$I_{SL\nu} = \frac{64}{3}\mu^6\gamma^4\left[\left(n_e\rho_e^{(1)}\vec{\beta}(1 - \vec{\beta}\vec{v}_e) - \frac{1}{\gamma}n_e\rho_e^{(1)}\vec{v}_{e\perp}\right)^2 + \vec{B}_\perp + \frac{1}{\gamma}\vec{B}_\parallel\right]^2, \tag{24}$$

where the terms proportional to γ^{-2} in the brackets are neglected. The spin light emission rate in non-moving matter and in the absence of the magnetic field is

$$\Gamma_{SL\nu} = \frac{\sqrt{2}}{3}\gamma^2\mu^2 G_F^3 n_e^3. \tag{25}$$

The $SL\nu$ in matter must be important for environments with high effective densities, n, because the total radiation power is proportional to n^4. The total power of the $SL\nu$ is increasing with the neutrino energy increase and is proportional to the fourth power of the neutrino Lorentz factor, $I_{SL\nu} \sim \gamma^4$. It has been also shown [16] that the $SL\nu$ is strongly beamed in the direction of neutrino propagation and is confined within a small cone given by $\delta\theta \sim \gamma^{-1}$.

The average energy of photons of the spin light in matter is

$$\omega_{SL\nu} = \sqrt{2}G_F n_e \gamma^2. \tag{26}$$

For the density $n_e \geq 10^{30} cm^{-3}$ and neutrino with mass $m_\nu = 1\ eV$ and energy $p_0 = 10\ MeV$ the energy range of emitted photons could span up to gamma-rays. These properties of the $SL\nu$ in matter enables us to predict that this radiation should be important in different astrophysical environments (quasars, gamma-ray bursts etc) and in dense plasma of the early Universe.

Let us briefly discuss the properties the $SL\nu$ produced by the neutrino spin evolution in gravitational fields like that of rotating neutron stars and black holes. If we consider [18] the neutrino propagating along the rotation axis of a massive object then the photon energy (in the laboratory frame) of the $SL\nu$ in the gravitational field of an object can be estimated as

$$\omega_{SL\nu} \sim \omega_0 \gamma \sim \frac{G_N L}{r^3} \gamma^2, \tag{27}$$

where L is the angular momentum of a rotating object, and G_N is the Newton's constant. If the angular momentum is chosen to be equal to the maximal allowed value $L = r_0^2/(4G_N)$ (see, for instance, [22]), where r_0 is the Schwarzschild radius, then for the photon energy we have

$$\omega_{SL\nu} \sim 10^{-11} \times \gamma^2 \left(\frac{r_0}{r}\right)^3 \text{eV}. \tag{28}$$

It follows that $\omega_{SL\nu} \sim 10\,\text{GeV}$ for $\gamma \sim 10^{12}$ and $r \sim 10\ r_0$. Note that for the mass of neutrino $m_\nu \sim 1\,\text{eV}$ the condition $(\omega/p_0) \sim 10^{-2} \ll 1$ is still valid, i.e. the quasiclassical approach to the neutrino spin evolution can be applied in this case.

It should be noted that the whole developed approach to the neutrino spin evolution and oscillations in different background environments (including the predictions for the corresponding new effects) can be generalized, with a minor modification, for neutrinos with the flavour changing magnetic moments. These cases correspond to models of the Dirac neutrinos with non-diagonal magnetic moments or the Majorana neutrinos which could also have transitional (magnetic) moments.

4   Conclusion

We have developed the Lorentz invariant approach to the neutrino flavour and spin oscillations and on this basis studied **the four new effects**: i) the neutrino spin oscillations in various electromagnetic field configurations, ii) the possibility for neutrino spin oscillations to be produced by weak interactions with matter and by gravitational fields of rotating massive objects, iii) the significant change in the neutrino (spin and flavour) oscillations pattern due to the relativistic motion of matter, and iv) the spin light radiation by a neutrino moving in matter and/or electromagnetic and gravitational fields (the *spin light of neutrino*, $SL\nu$, in background environments). We believe that **the four new effects** could have important consequences in different astrophysical and cosmological environments.

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